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An analog method for the determination of the deflection of beams resting on end supports and design of an automatic programming device

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AN ANALOG METHOD FOR THE DETERMINATION OF THE DEFLECTION OF BEAMS RESTING ON END SUPPORTS AND DESIGN OF AN AUTOMATIC PROGRAMMING DEVICE

BY

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A

THESIS

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Abstract

This investigation shows that beam problems can be solved easily and accurately with an analog computer. Two problems were solved as to their shear, moment, slope and deflection along the length of the beam and the graphs drawn by the analog computer were compared with calculated values. Very little difference was found between them.

An electronic programmer was built to aid in handling beam problems as they were being run on the computer. This was capable of putting into the computer bits of data at specified intervals of time. The programmer proved reliable and repeatable once it was adjusted to the time desired.

Acknowledgement

The author wishes to express his appreciation to Dr. T. R. Faucett for the idea behind this thesis and for his helpful cooperation rendered throughout the course of this investigation.

Table of Contents

List of Figures

List of Tables

List of Symbols

c	Proportionality Factor	in/sec
C_f	Capacitance	Ufd
C_1, C_2, C_3	Constants of Integration	
e	Voltage	Volt
e_i	Input Voltage	Volt
e_{io}	Constant of Integration	Volt
e_o	Output Voltage	Volt
e_1	Voltage Analogous to M	Volt
e_2	Voltage Analogous to θ	Volt
e_3	Voltage Analogous to y	Volt
E	Modulus of Elasticity	PSI
i	Electrical Current	Amperes
I	Moment of Inertia	in^4
k	Proportionality Factor	Volt/in
M	Moment	in-lb
M_o	Initial Moment	in-lb
P	Potential	Volt
Q	Charge on Capacitor	Coulombs
R_i	Resistance	Ohm
S	Distance	in
t	Time	sec
θ	Slope	Rad.
θ_o	Initial Slope	Rad.
v	Velocity	in/sec
V	Shear	psi

V_o	Initial Shear	psi
W	Load	lb/in
x	Distance along Beam	in
y	Deflection	in
y_o	Initial Deflection	in

I. INTRODUCTION

The solution of a problem by simple integration can easily be done on an analog computer by putting the prolem into a high gain amplifier which has resistive input and feedback through a capacitor. The solution to the differential equation can be obtained from the output of the amplifier. By connecting several of these amplifiers in series successive integration can be achieved.

The analog computer is ideal for the solution of beam problems. The third derivative of the deflection with respect to the distance along the beam is associated with the shear on the beam. One integration of this equation gives the moment, a second gives the slope and a third successive integration gives the deflection at any point along the beam.

The purpose of this thesis is to show how two beams resting on end supports and carrying combinations of uniform and concentrated loading can be solved on an analog computer. The purpose also is to demonstrate an electronic programming device which is essential for the ease of handling the problem while it is being solved on the computer.

The solving of beam problems with any complexity of loading is normally a process of many calculations. There needs to be a sufficient number of calculations at spaced intervals along the beam in order to be able to graph the moment, slope, or deflection curves with accuracy. If the method of superposition is used then the number of calculations along the beam is multiplied by the number of separate loads. The digital computer has been used quite extensively for this purpose. After calculating a sufficient number of deflections along the beam, it is then necessary to plot and graph the curves.

On the analog computer the job is done much more easily. After the problem is set up on the computer, the curves of shear, moment, slope and deflection are automatically drawn on graph paper. This can be accomplished in a few minutes.

The particular analog computer used for this work is small and limited in its range. It is normally used as an educational device. The indications are that any work done so far on it can be done more easily and accurately on a larger machine. In addition, beams loaded more complexly could be handled on a larger machine whereas they could not be done at all on this computer.

II. REVIEW OF LITERATURE

No literature was found in which this method of solving beam problems was used. There were many uses made of a digital computer to solve the problem by means of moment distribution and by use of slope deflection equations. In this case the beam was divided into ten equal intervals along its length and the moment and deflection were calculated at each interval.[1,2,3]

One paper described the use of an analog computer to determine the deflection at various points along a beam. This was done by setting up an electrical analogy for the beam by using resistors, capacitors or transformers to represent the properties of the beam. The solution of the problem was obtained by taking measurements at points in the circuit. These measurements correspond to the shear, moment, slope or deflection of the beam at that point.[4] The same had been applied to an arch dam analysis.[5]

Analog computers have been used to determine the frequency of vibration of beams. The deflections were found by first calculating influence coefficients which were used to find the vibration frequencies.[6]
(References cited here appear at the end of this thesis.)

III. ANALYSIS

A. Mathematical Analysis of a Beam in Bending

The differential equation of the elastic curve of a beam in bending is given as

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \tag{1}$$

in which y is the deflection of the beam, x is the distance along the beam, M is the moment at any point, E is the modulus of elasticity and I is the moment of inertia.

By differentiating once with respect to x and holding EI constant

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{M}{EI}\right) = \frac{1}{EI}\frac{dM}{dx} = \frac{V}{EI} \tag{2}$$

where V is the shear.

By differentiating again, the uniform load was obtained.

$$\frac{d^4y}{dx^4} = \frac{1}{EI}\frac{dV}{dx} = \frac{W}{EI} \tag{3}$$

If in the solving of a beam problem, the problem is started with the load and integrated four times, the following is obtained:

$$V = \int W dx + V_o$$

which is shear

$$M = \int V dx + M_o$$

which is moment (4)

$$\theta = \int \frac{M}{EI}\, dx + \theta_o$$

which is slope

$$y = \int \theta dx + y_0$$

which is deflection.

In the last two equations, E and I are involved.

In applying this to an analog computer, the problem can be started by putting in the shear along the beam and as a result the initial constant V_0 becomes the first shear constant entered into the computer. If there is an initial moment not equal to zero, then the computer can be adjusted until the initial moment is accounted for in the solution. The initial slope can be determined on the computer by a simple method of adjusting the initial slope input until the deflection curve starts and ends at the proper end deflections or adjusting the base line so that the same thing is accomplished.

The computer integrates with respect to time, therefore a relationship between time and the distance along the beam must be established. This relationship is linear

$$x = ct \tag{5}$$

due to the fact that the graph paper moves at a constant velocity. The relationship could also be written as

$$x = c\left(\frac{s}{v}\right) \tag{5a}$$

where s is the distance traveled and v is the velocity of travel.

The differential equation then is

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \frac{1}{c}\frac{dy}{dt} \tag{6}$$

If a relationship between voltage and deflection is set up which is linear

$$e = ky \tag{7}$$

then:

$$\frac{dy}{dt} = \frac{1}{k}\frac{de}{dt} \tag{8}$$

$$\frac{dy}{dx} = \frac{1}{ck}\frac{de}{dt}$$

$$\frac{d^2y}{dx^2} = \frac{1}{c^2k}\frac{d^2e}{dt^2}$$

$$\frac{d^3y}{dx^3} = \frac{1}{c^3k}\frac{d^3e}{dt^3} \tag{9}$$

$$\frac{d^4y}{dx^4} = \frac{1}{c^4k}\frac{d^4e}{dt^4}$$

It can be seen then that when a problem is set up starting with the third or fourth derivative that the ratio of the analogous computer values to the actual values is equal to k and to c to some power. Therefore, if the ratio of x to t is one to one such as one inch per second, then c equals one and need not be considered in the problem.

By initially starting with the third derivative which is shear, the problem is set up by equating the different values of shear equal to a starting potential P

$$\frac{d^3e}{dt^3} = P$$

Integrating this gives

$$\frac{d^2e}{dt^2} = Pt + C_1$$

$$\frac{de}{dt} = \frac{Pt^2}{2} + C_1t + C_2 \qquad (10)$$

$$e = \frac{Pt^3}{6} + \frac{C_1t^2}{2} + C_2t + C_3$$

From equation 2;

$$\iiint \frac{d^3y}{dx^3} = y = \frac{Vx^3}{6EI} + \frac{M_ox^2}{2} + \theta_ox + y_o \qquad (11)$$

The proportions between each term of equation 11 and the corresponding terms of the last equation of 10 are equal, therefore

$$\frac{y}{e} = \frac{\frac{Vx^3}{6EI}}{\frac{Pt^3}{6}} = \frac{Vx^3}{PEIt^3} = \frac{1}{k} \qquad (12)$$

$$\frac{V}{EI} = \frac{P}{kc^3} \qquad (13)$$

and the result is the starting relationship.

This type of computation then yields voltages at specific times that are related to shear, moment, slope, and deflection at corresponding distances as shown by the following relations

$$x = ct$$

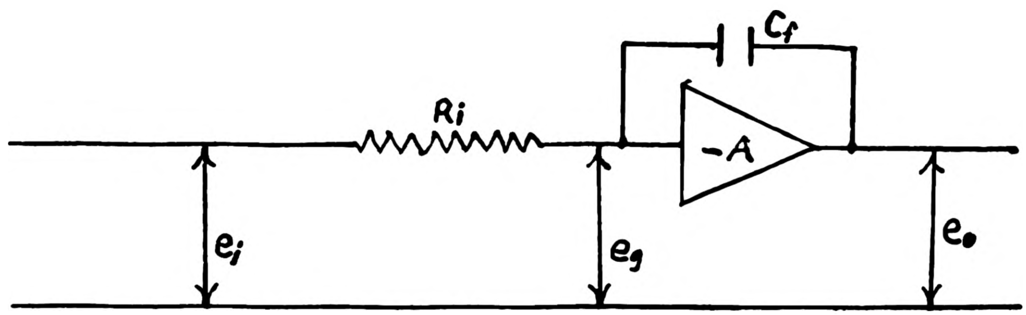
$$\frac{V}{EI} = \frac{P}{kc^3}$$

$$\frac{M}{EI} = \frac{e_1}{kc^2} \qquad (14)$$

$$\theta = \frac{e_2}{kc}$$

$$y = \frac{e_3}{k}$$

B. Analysis of the Integrating Circuit of the Computer



It is assumed that the gain of the amplifier is high and there is no grid current then

$$\frac{e_i}{R_i} = i = \frac{dQ}{dt}$$

Where e_i is the input voltage, R_i a resistor, i the current and Q a charge on the capacitor.

In addition there is the relationship

$$dQ = C_f de_o$$

Where C_f is the capacitance of the capacitor and de_o is the change in output voltage.

It can be seen that

$$\frac{e_i}{R_i} = + C_f \frac{de_o}{dt}$$

and solving for de_o,

$$de_o = \frac{+ 1}{R_i C_f} e_i dt$$

$$e_o = + \frac{1}{R_i C_f} \int e_i dt + e_{io}$$

Where e_{io} is the voltage across the capacitor at t equal to zero.

Thus it can be seen that the output voltage is the integral of the input voltage with respect to time.

When a constant voltage e_i is fed into the integrating circuit it will be amplified as it goes through the amplifier and will be present at the output. In the meantime the capacitor is discharging into the output combining with the original constant voltage. This variable is to the first power. If this is fed into a second integrating circuit this voltage will combine with the discharge from the second capacitor forming a curve in which the variable is to the second power. And so on to the third, fourth, etc. This is analogous to the results obtained when successive integrations are performed, starting with a constant. In this way an electrical analogy can be produced for an integrating process.

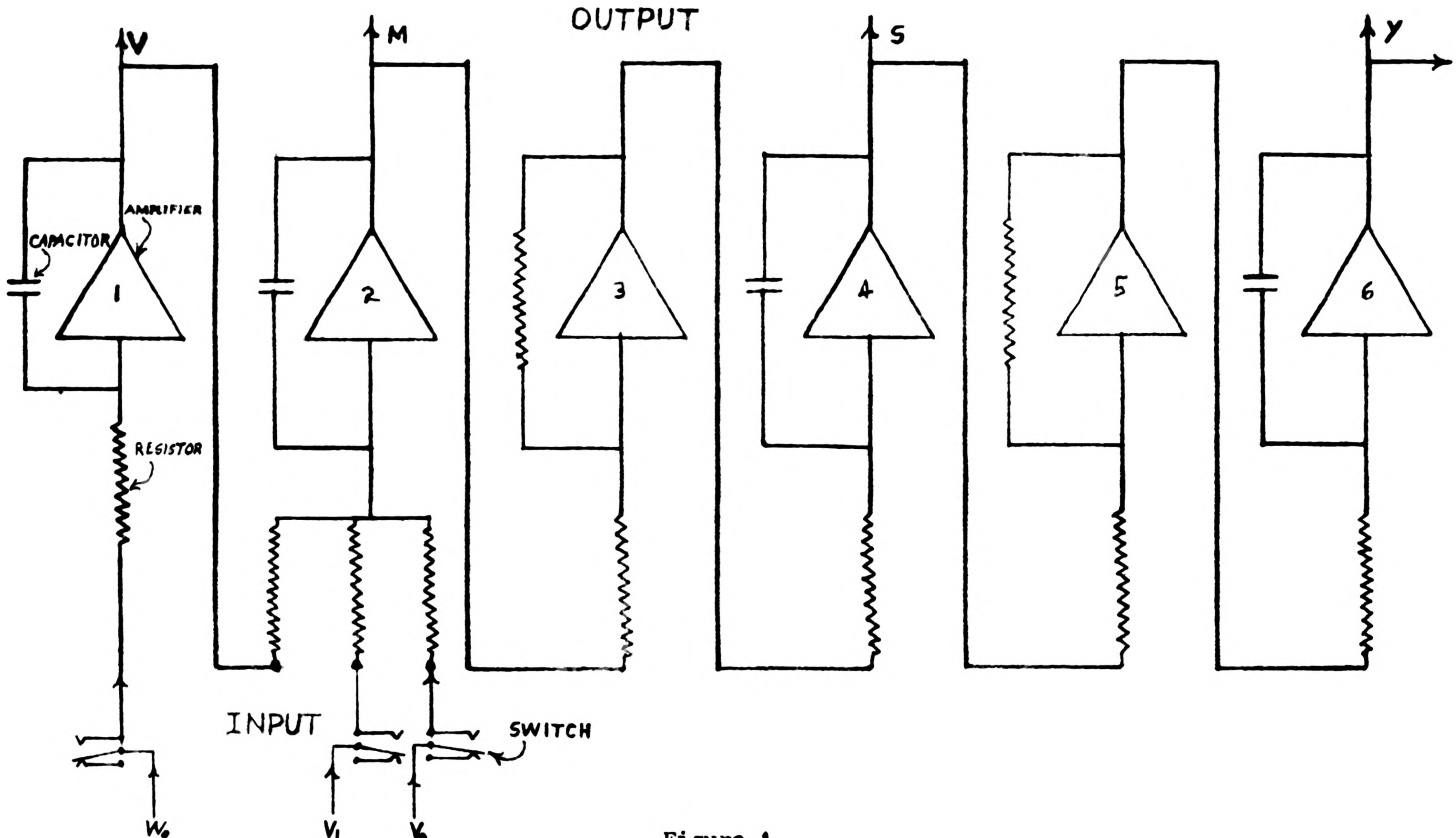


Figure 1

SCHEMATIC OF INTEGRATING CIRCUIT

C. Analysis of the Programmer Circuit

A programmer is required in order to introduce into the circuit of the computer certain constants at specified times during which a problem is being solved. The basic elements of the timing circuit are a capacitor which is connected to both the grid of a control tube and a variable resistor. The basic principle of the circuit concerns the time required for a charge on the capacitor to leak off through the resistor in sufficient quantity to reduce the voltage on the grid enough to allow the tube to conduct.

The capacitor is charged by shorting the terminal at w on the circuit diagram, figure number nine. AC current flows in a short loop through variable resistor g and then up onto the grid. The grid rectifies the current and charges capacitor b on the side next to the grid. A larger loop of current in the meantime flows around through the tube and energizes the solenoid which actuates the switches. When the short at w is broken, capacitor b starts discharging through the variable resistor d. While discharging, the capacitor maintains a voltage on the grid which has cut off the conduction of the tube. When the tube is not conducting, the circuit through the solenoid is broken and the switches are in their relaxed position. As soon as the grid voltage decreases to a certain low value, the tube will start conducting and the switches will be thrown.

As the programmer is set up, each pair of terminals are open when the switch is relaxed and becomes shorted when the switch is actuated. The time between the opening of terminal w and the closing of the switches again is controlled by the variable resistor g.

IV. Description and Setup of Apparatus

The Heath Educational Analog Computer has nine integrating circuits. Each integrating circuit is essentially an amplifier with feedback through a capacitor. For one of the beam problems three were used for purposes of integration and two for amplification. For the other problem involving the uniform load, four were used for integration and two as amplifiers. The voltages which were put into the computer were set by variable resistors of which three were used out of the five on the set. The set contains four relays which were thrown when the main start compute switch was thrown and as a result started all the elements of operation in the computer simultaneously. There are three initial condition controls which can be used to set initial shear, moment and slope in a problem. The input was controlled by an electronic programmer. The output was to a Brush Recorder.

The programmer contains five independent 4 pole double throw switches. These switches can be thrown automatically at any increment of time from 0 to 60 seconds. A usable circuit is completed only when the switches have been thrown. The capacitors in the programmer are charged by shorting terminals located in the back and is controlled by a switch located in the computer. When the computer switch is turned on to start the solving of a problem, the charging circuit is broken.

The Brush Recorder obtains its input from the output amplifier of the computer. It holds a roll of graph paper which can be moved across recorder pens at controlled speeds of 1, 5, 25 and 125 millimeters per second. There are two pens, each of which can cover one half the width of the graph paper. These are capable of drawing continuous curves as the graph paper is moved across them. Their normal position is in the middle of their respective half of the paper and can be adjusted in a lateral direction by a pen bias control. Each pen is independent of the other and has its own separate controls. The pens are able to move in either direction perpendicular to the direction of motion of the paper depending on the polarity of the voltage applied. The distance moved depends on the magnitude of the voltage and the voltage per chart line control. This control is a ten position switch varying from .01 to 10 volts per chart line. There are twenty chart lines on either side of the neutral middle line which means that a maximum of 200 volts can be recorded. The computer, however, is capable of putting out only 50 volts.

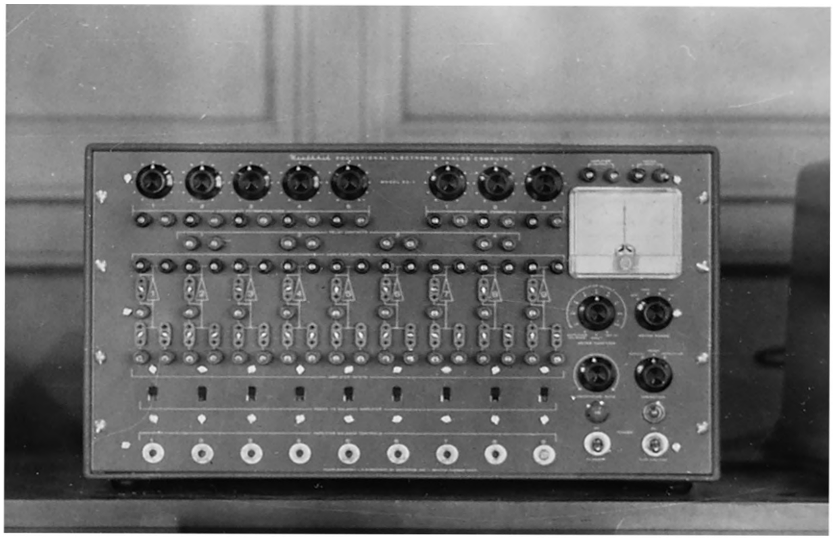


Figure 2. The Analog Computer

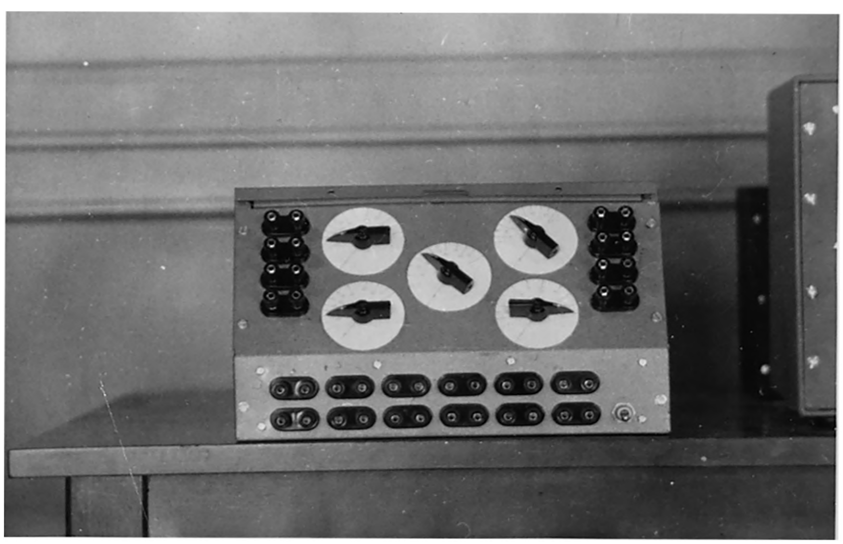


Figure 3. The Electronic Programmer

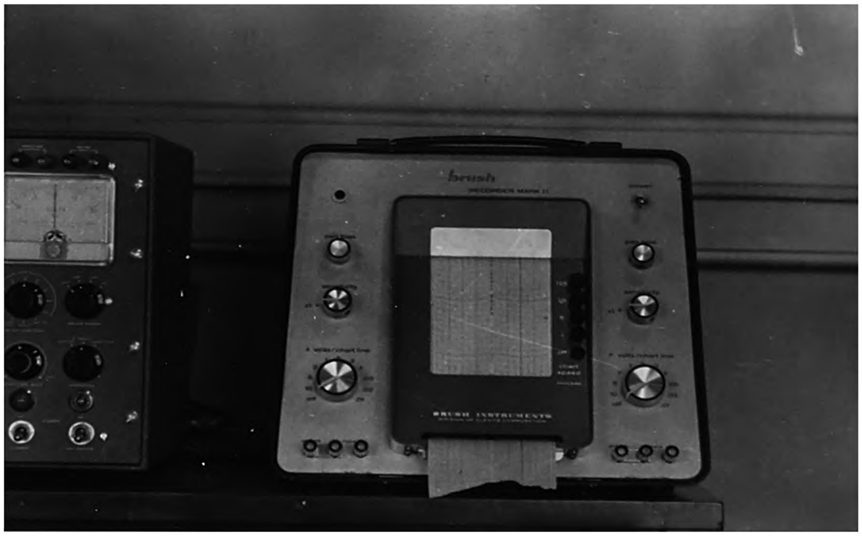


Figure 4. The Brush Recorder

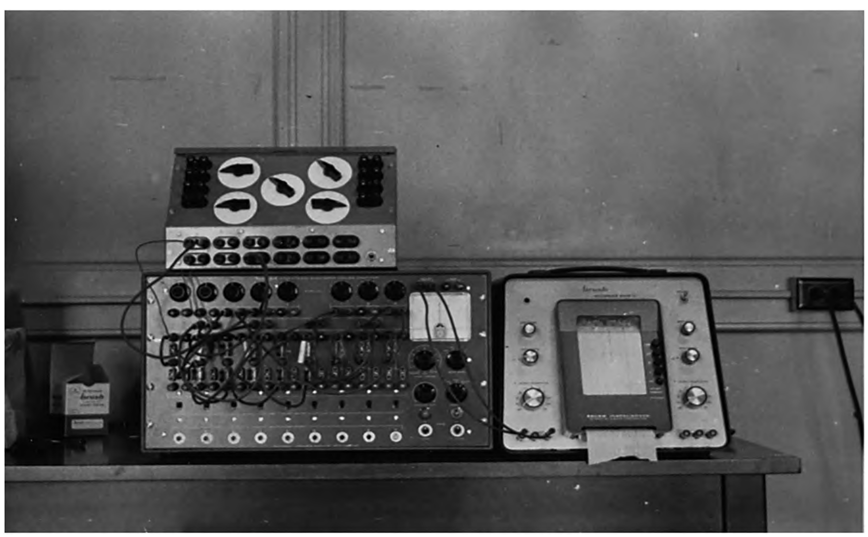


Figure 5. A Set Up for Solving a Problem

V. Procedure for Setting Up a Problem

First of all, the programmer is set to throw its switches at the proper time. This can be done by using the recorder to show where the switches are actuated by changes in the level of the curve. Next the capacitor on the amplifier which integrates the shear curve is replaced by a resistor. With the switch set to take the output off that integrator, the compute start switch is turned on and the voltage level controls are adjusted to give the correct level of the shear curve on the graph. After this is set correctly, the resistor is replaced by the capacitor. The output of each of the integrators can be switched to the recorder and in this way the moment, slope, and deflection curves can each be drawn.

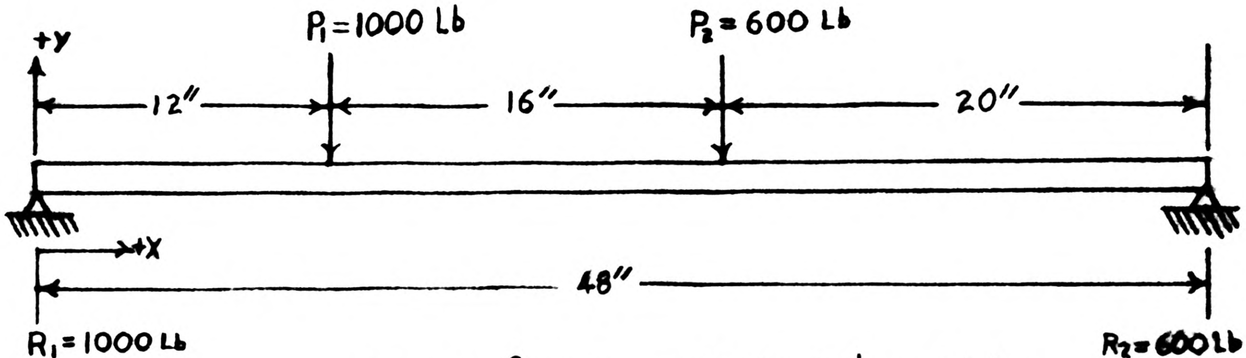
VI. Results

The results obtained by calculation and by use of the computer compare favorably. In order to obtain these results, careful setting up of the shears and timing were required.

When the shears were set up a stray 60 cps AC voltage was found to be on the DC. This is shown by the heavy lines on the shear graphs. The middle of each of these heavy lines was taken as the average. After an integration the AC voltage disappeared. This was due to the fact that when a sine wave is integrated over one complete cycle, the sum of the areas equal zero. Another problem was the fact that when the start compute switch was turned on, the relays in the programmer were about a quarter second slow in reacting. The result was little or no area under the shear curve to integrate during that time. This started the integration late and caused the curves to be slightly lower than they should have been. This can be corrected by lowering the base of the curves.

In the running of the problem with the uniform load, an attempt was first made to start integrating from the end with the uniform load. This means that it was necessary to produce a linearly decreasing voltage for the shear along the uniform load, then stop the decrease and hold at a constant for the rest of the beam. A few attempts at this showed that the voltage became erratic. It was then decided to start at the end with the concentrated load and end up with the linearly varying voltage. The results were far more satisfactory.

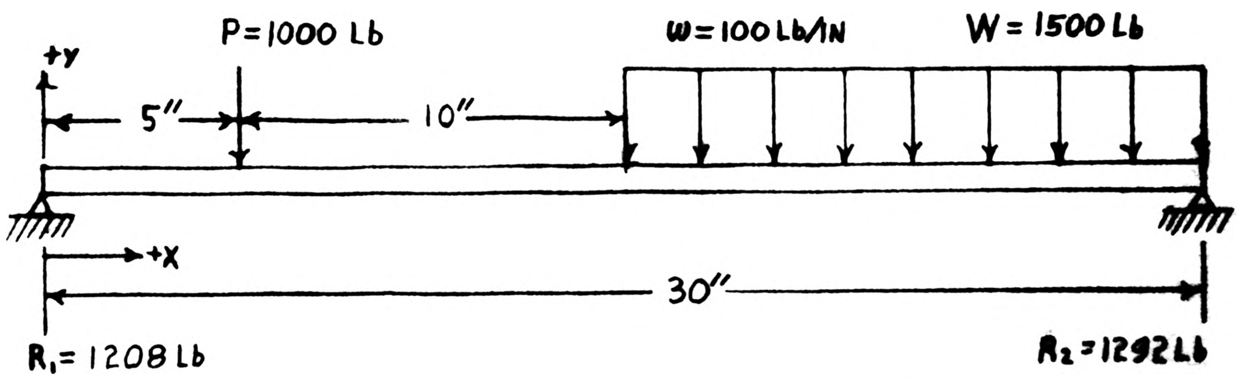
Once the programmer was adjusted to the time desired it proved very reliable and repeatable in the lower part of the scale. Not much use was made of it in the upper scale portion due to the fact that it would have produced long graphs and high voltages.



$E=30x10^6$ PSI $\quad$ $k=2.82x10^4$ volt/in

$I=6$ in^4 $\quad$ c=2 in/sec

(a) Beam No. 1



$E=30x10^6$ PSI $\quad$ $k=2x10^5$ volt/in

$I=5$ in^4 $\quad$ c=1 in/sec

(b) Beam No. 2

Figure 6. Beam Load Diagram

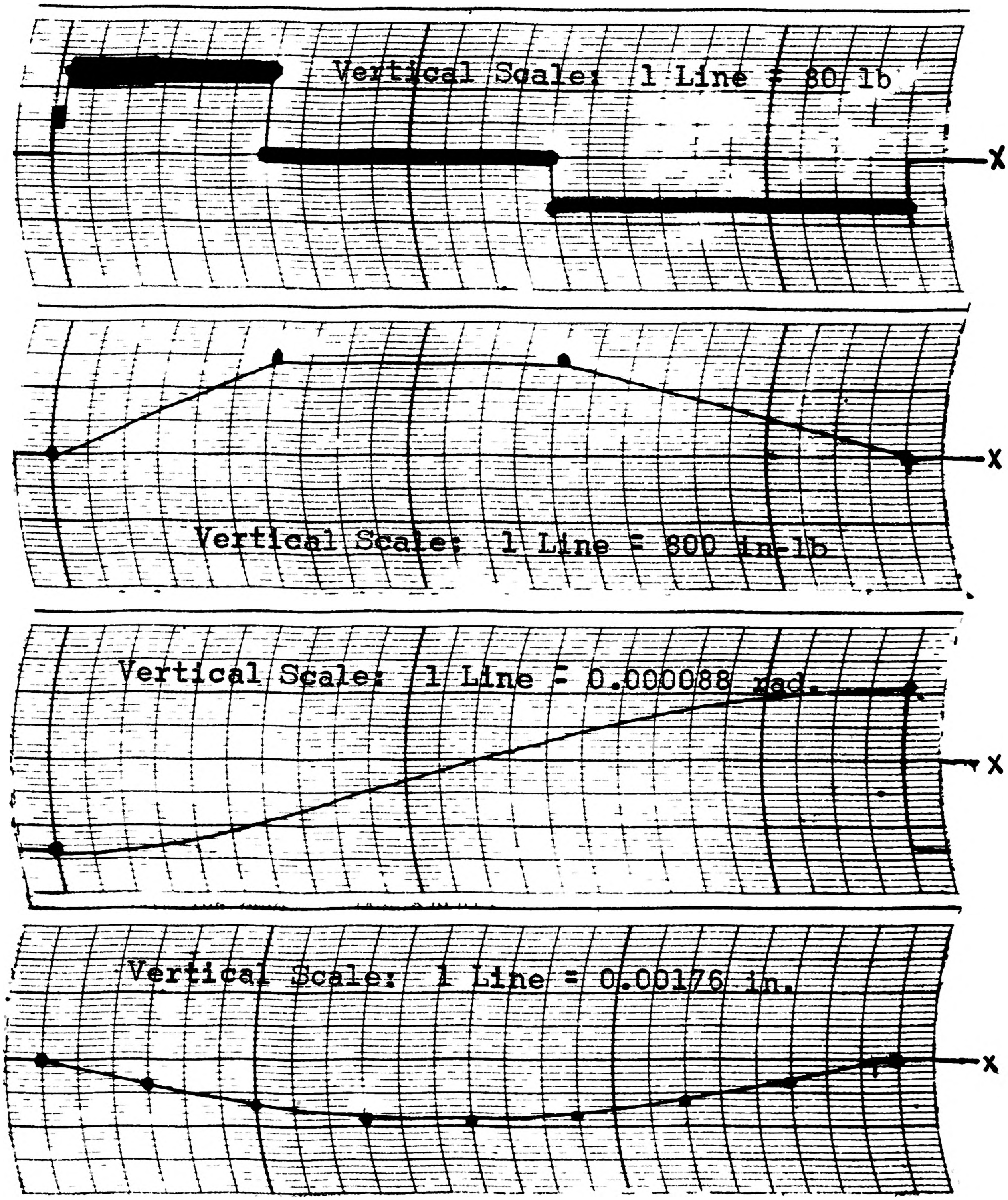


Figure 7

Graph of Beam No. 1 showing from top to bottom the shear, moment, slope and deflection curves drawn by the analog computer. The heavy dots are the values obtained by calculation.

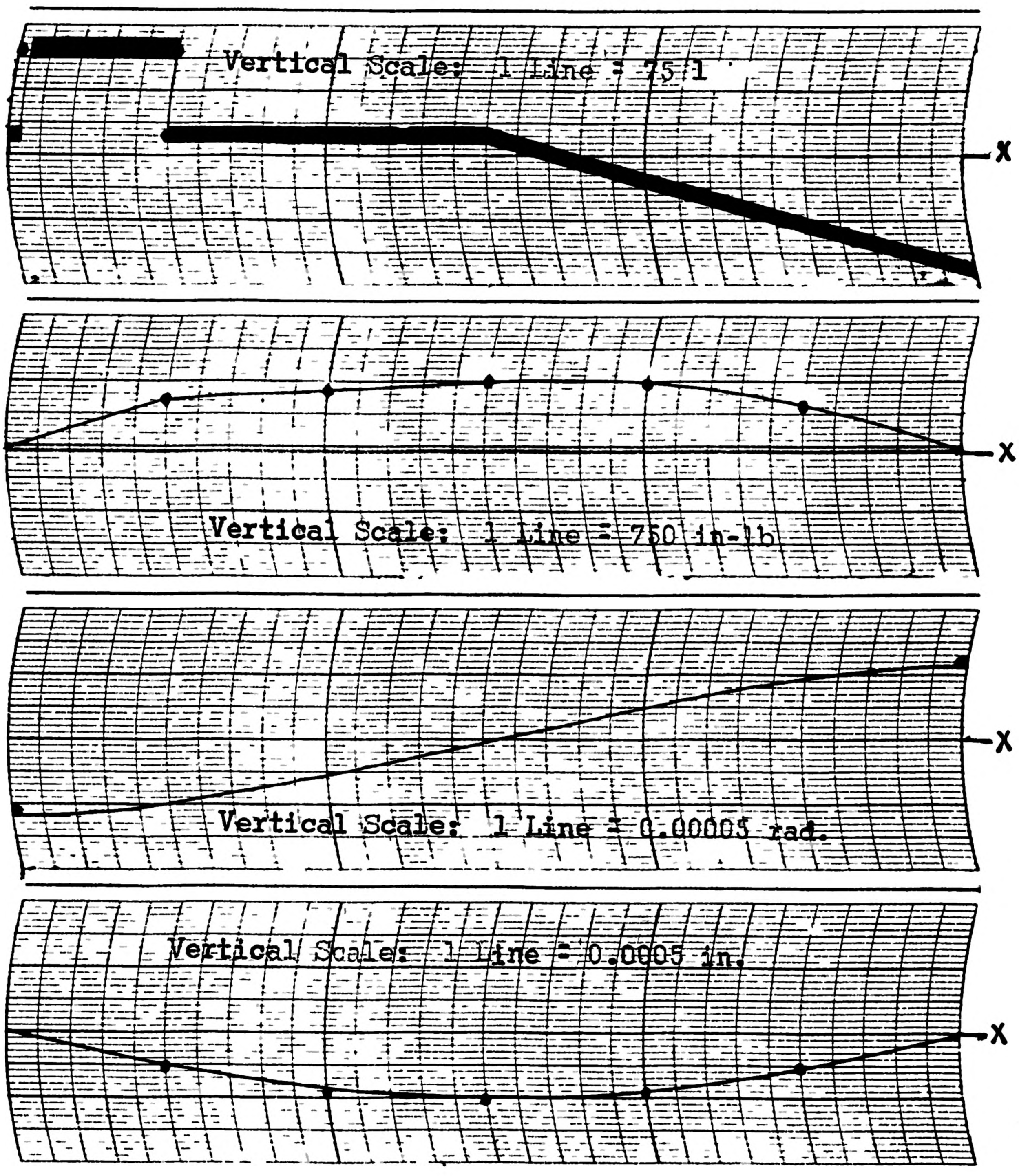


Figure 8

Graph of Beam No. 2 showing from top to bottom the shear, moment, slope and deflection curves drawn by the analog computer. The heavy dots are the values obtained by calculation.

VII. Conclusions

From the results it can be concluded that the analog computer can be used to solve beam problems of the type studied, that is any type resting on supports at each end, having a combination of uniform or concentrated loading and in which the section is constant. The results can be obtained easily and reliable. The computer can be set up to solve a series of beams of this type, the only additional work involved being that of setting up each shear diagram and adjusting the timing.

The indications are that the computer can also be used for solving beam problems of other types such as variable moment of inertia, variable modulus of elasticity, cantilever beams, etc.

The programmer has proved to be reliable and accurate to a few hundredths of a second. This means that on a beam problem which requires 30 seconds or more of continuous integration, the error introduced by the programmer will be insignificant.

VIII. Suggestion for Further Study

In the case of solving a beam for variable moment of inertia or modulus of elasticity, it is necessary to put in the V/EI curve, which has a different shape from the shear curve, and to introduce a change of curvature at the start of each new section.

A beam with a triangular loading can be solved by the introduction of another integrating circuit. This will produce a linearly varying voltage which is a first degree curve and when this is fed through the second integrator a second degree curve will be produced. This curve is the shear curve for the loading.

Another type is a cantilever beam with any type of loading or with couples. This requires the insertion of the shears and the initial moment. The beam types resting on end supports have an initial and final moment of zero. The cantilever beam has an initial moment if the integration is started at the fixed end. If started at the other end, the initial moment is zero and the final moment is some value not equal to zero.

Another type is one in which there is one or more intermediate supports between the end supports. If the reactions can be determined, then the shear diagram can be drawn and the shears can be used to solve the problem.

There are many others such as beams having both ends fixed or one end fixed and the other on a support. These can be solved as long as the shears can be found. Any shape of loading can be used as long as it can be approximated into a mathematical curve which when differentiated a certain number of times will become a constant. Then starting with that constant it can be integrated back to the original loading and beyond into the problem. It could be possible to use a function generator for this purpose.

Appendix

TABLE I

SHEAR:	X= 0-12"	12-28"	28-48"
CALCULATED	1000 Lb	0	-600
FROM GRAPH	1000 Lb	-5	-580

MOMENT:	X= 0	12"	28"	48"
CALCULATED	0 IN-LB	12,000	12,000	0
FROM GRAPH	0 IN-LB	11,800	11,700	0

SLOPE:	LEFT END	RIGHT END
CALCULATED	-0.00118 RAD	0.00096
FROM GRAPH	-0.00127 RAD	0.00095

DEFLECTION:	6"	12"	18"	24"	30"	36"	42"
CALCULATED	-0.006551 IN	-0.01215	-0.0154	-0.0163	-0.0148	-0.0110	-0.00589
FROM GRAPH	-0.00649 IN	-0.01190	-0.0144	-0.0156	-0.0142	-0.0107	-0.00560

CALCULATIONS FOR BEAM No 1

TABLE II

SHEAR:	0 - 5″	5 - 15″	15 - 30″
CALCULATED	1208 Lb	208	208 - 292
FROM GRAPH	1160 Lb	225	225 - 1330

MOMENT:	5″	10″	15″	20″	25″
CALCULATED	6040 IN-LB	7080	8120	7920	5220
FROM GRAPH	6000 IN-LB	7050	8100	7950	5400

SLOPE:	LEFT END	RIGHT END
CALCULATED	-0.000561 RAD	0.000588
FROM GRAPH	-0.000578 RAD	0.000565

D FLECTION:	5″	10″	15″	20″	25″
CALCULATED	-0.00271 IN	-0.00461	-0.00516	-0.00430	-0.002733
FROM GRAPH	-0.00250 IN	-0.00430	-0.00500	-0.00465	-0.002750

CALCULATIONS FOR BEAM No 2

Beam No. 2:

Slope:

$$\theta_{L.E.} = -\frac{1}{6}\frac{P}{EI}\left(bL - \frac{b^3}{L}\right) + \frac{1}{48EI}\left[16R_1L^2 - W\left(24d^2 - \frac{8d^3}{L} + \frac{b_1^3}{L}\right)\right]$$

$W = 1500\,Lb$, $b = 25''$ $R_1 = 1125\,Lb$ $d = 22.5''$ $b_1 = 15''$

$$\theta_{R.E.} = \frac{1}{6}\frac{P}{EI}\left(2bL + \frac{b^3}{L} - 3b^2\right) + \frac{1}{48EI}\left[8RL^2 - W\left(\frac{8d^3}{L} - \frac{b_1^3}{L} + 2b_1^2\right)\right]$$

Deflection:

$0'' - 5''$

$$y = -\frac{PbX}{6EIL}\left[2L(L-X) - b^2 - (L-X)^2\right] + \frac{1}{48EI}\Big\{8R_1(X_1^3 - L^2X_1) +$$

$$WX_1\left[\frac{8d^3}{L} - \frac{b_1^3}{L}\right] - 8W\left(X_1 - \tfrac{1}{2}a_1 - \tfrac{1}{2}b_1\right)^3 + W(b_1^3)\Big\}$$

$b = 25''$ $R_1 = 1125\,Lb$ $X_1 = L - X$ $b_1 = 15''$ $a_1 = 0$

$5'' - 15''$

$$y = -\frac{Pa(L-X)}{6EIL}\left[2Lb - b^2 - (L-X)^2\right] + \frac{1}{48EI}\Big\{8R_1(X_1^3 - L^2{}_1) +$$

$$WX_1\left[\frac{8d^3}{L} - \frac{b_1^3}{L}\right] - 8W\left(X_1 - \tfrac{1}{2}b_1\right)^3 + Wb_1^3\Big\}$$

$a = 5''$

$15'' - 30''$

$$y = \frac{-Pa(L-X)}{6EIL}\left[2Lb - b^2 - (L-X)^2\right] + \frac{1}{48EI}\Big\{8R_1(X_1^3 - L^2X_1 +$$

$$WX_1\left[\frac{8d^3}{L} - \frac{b_1^3}{L} + 2b_1^2\right] - 2W\frac{X_1^4}{b_1}\Big\}$$

Formulas Used For Slope And Deflection[7]

Beam No. 1:

Slope:

$$\theta_{L.E.} = -\frac{1}{6}\frac{P_1}{EI}\left(bL - \frac{b^3}{L}\right) - \frac{1}{6}\frac{P_2}{EI}\left(2b_1L + \frac{b_1^3}{L} - 3b_1^2\right)$$

$b = 36''$ $\quad b_1 = 28''$ $\quad L = 48''$

$$\theta_{R.E.} = \frac{1}{6}\frac{P_1}{EI}\left(2bL + \frac{b^3}{L} - 3b^2\right) + \frac{1}{6}\frac{P_2}{EI}\left(b_1L - \frac{b_1^3}{L}\right)$$

Deflection:

$0'' - 12''$

$$y = -\frac{P_1 bX}{6EIL}\left[2L(L-X) - b^2 - (L-X)^2\right] - \frac{P_2 a_1(L-X_1)}{6EIL}\left[2Lb_1 - b_1^2 - (L-X_1)^2\right]$$

$b = 36''$ $\quad a_1 = 20''$ $\quad b_1 = 28''$ $\quad X_1 = L - X$

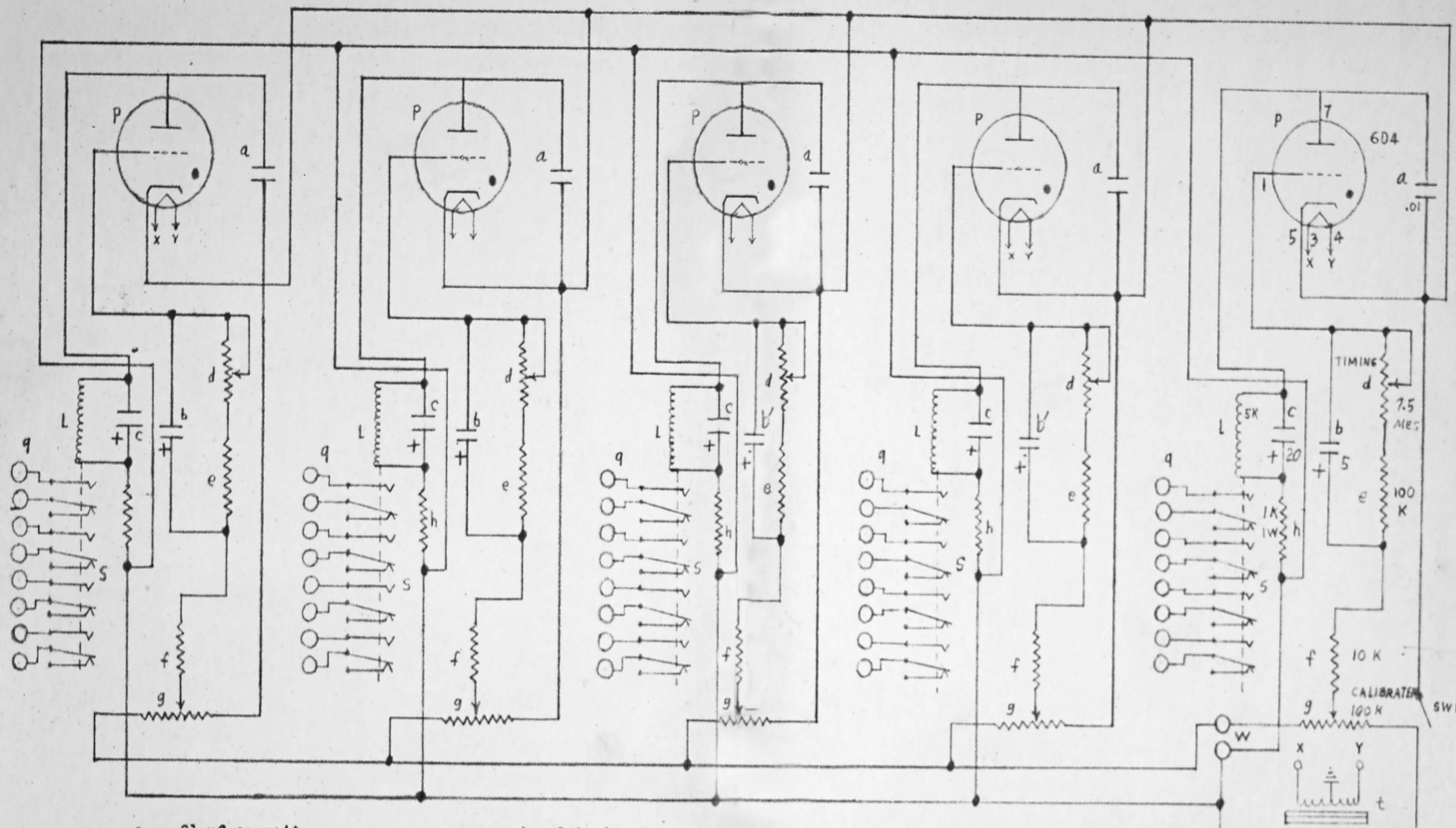
$12'' - 28''$

$$y = -\frac{P_1 a(L-X)}{6EIL}\left[2Lb - b^2 - (L-X)^2\right] - \frac{P_2 a_1(L-X_1)}{6EIL}\left[2Lb_1 - b_1^2 - (L-X_1)^2\right]$$

$a = 12''$

$28'' - 48''$

$$y = -\frac{P_1 a(L-X)}{6EIL}\left[2Lb - b^2 - (L-X)^2\right] - \frac{P_2 b_1 X_1}{6EIL}\left[2L(L-X_1) - b_1^2 - (L-X_1)^2\right]$$



a - .01 uf capacitor
b - 5 uf capacitor
b́ - 4 uf capacitor
c - 20 uf capacitor
d - 7.5 meg variable resistor
e - 100 k resistor
f - 10 k resistor
g - 100 k variable resistor

h - 1 k, 1 watt resistor
l - 5 k solenoid
p - 6d4 tube
q - plug in to switches

s - 4 pole double throw switches
t - transformer for filaments
w - plug in at back
sw1 - switch on back
sw2 - switch on front

Figure 9

ELECTRONICROGRAMMER CIRCUIT

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Vita

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